

# Quaternate generalization of Pfaffian state at $\nu = 5/2$

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(Dated: February 2, 2008)

We consider a quaternately generalized Pfaffian  $\text{QGPf}(\frac{1}{J(z_i, z_j, z_k, z_l)})[J(z_1, \dots, z_N)]^2$  in which the square of Vandermonde determinant,  $[J(z_1, \dots, z_N)]^2$ , implies the upmost Landau level is half filled. This wave function is the unique highest density zero energy state of a special short range interacting Hamiltonian. One can think this quaternate composite fermion liquid as a competing ground state of Moore-Read (MR) Pfaffian state at  $\nu = 5/2$ . The degeneracy of the quasihole excitations above the QGPf is higher than that of Moore-Read even Read-Rezayi quasiholes. The QGPf is related to a unitary conformal field theory with  $Z_2 \times Z_2$  parafermions in coset space  $SU(3)_2/U(1)^2$ . Because of the level-rank duality between  $SU(3)_2$  and  $SU(2)_3$  in conformal field theory, these quasiholes above this QGPf state obeying non-abelian anyonic statistics are expected to support the universal quantum computation at  $\nu = 5/2$  as Read-Rezayi quasiholes at  $\nu = 13/5$ . The edge states of QGPf are very different from those of the Pfaffian's.

PACS numbers: 73.43.-f, 71.10.Pm, 71.27.+a

**Introduction** — Fractional quantum Hall (FQH) states in the second Landau level exhibit a very complicated behavior because of the competition among many nearly degenerate states [1, 2]. There are two very interesting FQH states with  $\nu = \frac{5}{2}$  and  $\frac{12}{5}$  whose quasihole excitations are expected to obey the non-abelian anyonic statistics [3, 4], which is thought as the key to open the door to topological quantum computation [5, 6].

The  $\nu = \frac{5}{2}$  FQH state may possibly be explained by Moore-Read(MR) Pfaffian state [3]. It is two-electron cluster liquid state and can be thought as a  $p_x + ip_y$  weak pairing state of the composite fermions [7, 8]. The electron cluster states are not strange in a perpendicular magnetic fields. At higher Landau levels, say  $3 < \nu < 4$ , there are multi-electron bubble crystals away from the half filling. The electron number within a bubble may exceed two as the filling factor gradually goes to a half and a unidirectional charge density wave appears near the half filling [9]. Experimental evidence was already seen [10]. At  $\nu = 5/2$ , experiments found a transition from the FQH state to a unidirectional charge density wave when an in-plane magnetic field is applied [11].

Although it is widely believed that the FQH state at  $\nu = 5/2$  is a  $p$ -wave weak paired composite fermion state, none of experimental data confirms it. Moreover, if the  $\nu = 13/5$  and  $12/5$  FQH states are described by three-electron cluster Read-Rezayi state [4], the pairing state in  $\nu = 5/2$  seems to be contrary to the appearing order of number of electrons in a single cluster in higher Landau level in which the electron number in a single cluster increases as the filling factor closes to the half filling. The physical origin of these electron clusters appearing is the same: The lower fully filled Landau levels' screening supplies an effective attraction between electrons in the partially filled upmost Landau level. Therefore, one may raise a question: Is it possible that the electron number in a cluster exceeds two at  $\nu = 5/2$ ?

Numerically, although Morf gave a nice evidence that the MR Pfaffian state is energetically favored comparing to other competing states [12], only the even total number of electrons was examined because MR state pairing state is for even number of electrons. For the odd total electron number, the extra unpairing electron restricts the boundary condition of the finite system [8] and there is no numerical study.

It was known that the non-abelian statistics of MR quasiholes is not sufficiently dense for a universal quantum computer [6]. The Read-Rezayi quasiholes are dense to realize the universal quantum computing but the FQH state at  $\nu = 13/5$  (or  $12/5$ ) is more delicate. Is it possible that the quasiholes of the multi-electron clustered state at  $\nu = 5/2$  provide a base of the universal quantum computation? This is another motivation to consider the 4-electron cluster state at  $\nu = 5/2$ .

In this paper, we propose a quaternately generalized Pfaffian (QGPf) state if the electron number is integer times of 4. The pairing picture of composite fermions for the Pfaffian is naturally generalized to quaternate composite fermions. We find that the QGPf state may be the unique highest density zero energy state of a special Hamiltonian with short range interaction. The quasihole wave functions are the higher flux zero energy states of the special Hamiltonian. The quasihole degeneracy is much higher than that of MR even Read-Rezayi quasiholes. The QGPf state is related to a  $c = 6/5$  unitary conformal field theory(CFT) with  $Z_2 \times Z_2$  parafermions in coset space  $SU(3)_2/U(1)^2$  [13]. Due to the level-rank duality between  $SU(3)_2$  and  $SU(2)_3$  in CFT, these quasiholes obeying non-abelian anyonic statistics are expected to support the universal quantum computation. We also discuss the edge states of the QGPf and find that they are very different from those of Pfaffian.

**QGPf Wave Functions** — We consider two-dimensional spin polarized electron gas in the second Landau level.

The mixing between Landau levels is neglected and the second Landau level is treated as the lowest Landau level (LLL) except the interaction between electrons is renormalized due to the screening of the electrons in the LLL. We focus on the half filling, i.e.,  $\nu = 5/2$ . For even number of electrons, we recall the MR Pfaffian state [3], e.g., for 8-electrons, which is given by

$$\begin{aligned} & \mathcal{S}[(z_1 - z_3)(z_1 - z_4)(z_1 - z_5)(z_1 - z_6)(z_1 - z_7)(z_1 - z_8) \\ & (z_2 - z_3)(z_2 - z_4)(z_2 - z_5)(z_2 - z_6)(z_2 - z_7)(z_2 - z_8) \\ & (z_3 - z_5)(z_3 - z_6)(z_3 - z_7)(z_3 - z_8) \\ & (z_4 - z_5)(z_4 - z_6)(z_4 - z_7)(z_4 - z_8) \\ & (z_5 - z_7)(z_5 - z_8)(z_6 - z_7)(z_6 - z_8)]J(z_1, \dots, z_8) \\ & = \text{Pf}\left(\frac{1}{z_i - z_j}\right)[J(z_1, \dots, z_8)]^2 \end{aligned} \quad (1)$$

where  $\mathcal{S}$  denotes the symmetrization of  $1, \dots, 8$  and  $J(z_1, \dots, z_8) = \prod_{i < j \leq 8} (z_i - z_j)$  is the Vandermonde determinant.  $\text{Pf}\left(\frac{1}{z_i - z_j}\right) = \mathcal{A}\left(\frac{1}{(z_1 - z_2)(z_3 - z_4)(z_5 - z_6)(z_7 - z_8)}\right)$  with  $\mathcal{A}$  denoting the anti-symmetrization. For simplicity, we have omitted the Gaussian factor of the wave function.

The Read-Rezayi state [4] is a generalization of the MR Pfaffian. This wave function is for electron number  $3n$  and is a possible competing ground state of spin polarized electron gas at  $\nu = 13/5$ . There is another generalization of the MR state to  $N = 4n$ -electrons ( $n > 1$ ), say  $N = 8$ ,

$$\begin{aligned} & \mathcal{S}[(z_1 - z_5)(z_1 - z_6)(z_1 - z_7)(z_1 - z_8) \\ & (z_2 - z_5)(z_2 - z_6)(z_2 - z_7)(z_2 - z_8) \\ & (z_3 - z_5)(z_3 - z_6)(z_3 - z_7)(z_3 - z_8) \\ & (z_4 - z_5)(z_4 - z_6)(z_4 - z_7)(z_4 - z_8)]J(z_1, \dots, z_8). \end{aligned} \quad (2)$$

The symmetrizing part of this wave function allows 4-particles occurring the same position but not 5-particles. The Vandermonde determinant  $J(z_1, \dots, z_8)$  leads to the total wave function is anti-symmetric and electrons obey Pauli principle. This state may be rewritten as  $\text{QGPF}\left(\frac{1}{J(z_i, z_j, z_k, z_l)}\right)[J(z_1, \dots, z_8)]^2$  where the generalized Pfaffian (QGPf) is defined by  $\mathcal{A}\left(\frac{1}{J(z_1, z_2, z_3, z_4)J(z_5, z_6, z_7, z_8)}\right)$  and  $J(z_i, z_j, z_k, z_l)$  is the Vandermonde determinant for  $z_i, z_j, z_k$  and  $z_l$ . Generalizing to the system with  $N = 4n$  ( $n > 1$ ) electrons, we have

$$\text{QGPf}\left(\frac{1}{J(z_i, z_j, z_k, z_l)}\right)[J(z_1, \dots, z_N)]^2, \quad (3)$$

where the QGPf is defined by  $\mathcal{A}\left(\frac{1}{J(z_1, z_2, z_3, z_4)} \cdots \frac{1}{J(z_{N-3}, z_{N-2}, z_{N-1}, z_N)}\right)$ . The filling factor of this state is,  $\nu = \frac{N}{N_\phi}$  for  $N_\phi = 2(N-1) - 3$ , which tends to  $1/2$  as  $N \rightarrow \infty$  and coincides with  $\nu = 2 + 1/2$  in the second Landau level.

*Special Hamiltonian* — It was known that the MR Pfaffian state is the highest density zero energy state of the

Hamiltonian  $H_{MR} = V \sum_{i < j < k} \delta'(z_i - z_j) \delta'(z_j - z_k)$  [7]. The Hamiltonian, whose highest density zero energy state is the Read-Rezayi state, is given by  $H_{RR} = V \sum_{i < j < k < l} \delta'(z_i - z_j) \delta'(z_j - z_k) \delta'(z_k - z_l)$ . Therefore, the MR Pfaffian state and Read-Rezayi state are the corresponding ground states of these special Hamiltonians [4], according to Haldane's highest density criteria [14].

The QGPf is zero energy state of  $H_{RR}$  but not the highest density one. It is not zero energy state of  $H_{MR}$ . Can the QGPf state be a ground state of a special short range interacting Hamiltonian? For  $N = 4n$  electrons, we consider the following Hamiltonian

$$\begin{aligned} H = V \sum_{P_{4n}} & [\delta'(z_{i_1} - z_{i_2}) \delta'(z_{i_2} - z_{i_3})] \cdots \\ & [\delta'(z_{i_{4n-3}} - z_{i_{4n-2}}) \delta'(z_{i_{4n-2}} - z_{i_{4n-1}})] \\ & \delta'(z_{i_{4a}} - z_{i_{4b}}), \end{aligned} \quad (4)$$

where  $P_{4n}$  is a permutation of  $1, \dots, 4n$  with  $i_1 < i_2 < i_3; \dots; i_{4n-3} < i_{4n-2} < i_{4n-1}$ , and  $i_{4a} < i_{4b}$  with  $a \neq b \leq n$ . Here, we divide electrons into  $n$ -groups with 4 electrons in each group. Take three in each group and let them interacting with a three-body short range potential. Left electrons belong to the distinct groups and the last pair in the Hamiltonian comes from them. Then make all these electrons interacting simultaneously. This  $H$  outwardly is a  $3n + 2$ -electron interaction and in principle can be treated by means of the method developed in a recent work by Simon et al [15] but it is hard to handle when electron number becomes large. However, since it has been fractionized to independent  $n$ -three-body interaction and a two-body interaction, this special form of the interaction here in fact reflects the three-body interaction physics and it helps us to attract the lowest flux (i.e., the highest density) zero energy state. Taking  $N = 8$  as an example, the Hamiltonian is given by

$$\begin{aligned} H_8 = V & [\delta'(z_1 - z_2) \delta'(z_2 - z_3) \delta'(z_5 - z_6) \delta'(z_6 - z_7) \\ & \delta'(z_4 - z_8) + \text{other terms by cycling } (1 \cdots 8)], \end{aligned} \quad (5)$$

The zero energy wave function is written as  $\Psi_{\text{electron}}(z_1, \dots, z_8) = \Psi_{\text{symm}}(z_1, \dots, z_8) J(z_1, \dots, z_8)$ . If we consider only the symmetric part  $\Psi_{\text{symm}}$ , the  $\delta'$ -function should be replaced by the  $\delta$ -function. In order to find the lowest flux zero energy state, we divide eight electrons into two groups, say, (1234) and (5678). The relevant terms in the Hamiltonian are the terms including an inter group pair, say the pair  $z_4 - z_8$  in the first term of  $H_8$ . Thus, the most economic way to get the lowest flux is to include only this pair in  $\Psi_{\text{symm}}$  which then includes a term  $\prod_{i,j=1}^4 (z_i - z_{4+j})$ . One can check that all other terms in (5) (for  $\delta$ -function) act on it vanishing and if taking away any factor from it, one can always have a non-zero acting. Therefore, this is a lowest flux zero energy state. When regrouping the electrons, the lowest flux state also

changes correspondingly. Due to the total symmetry of  $\Psi_{\text{symm}}$ , regrouping leads to a unique lowest flux state, i.e.,  $\Psi_{\text{symm}}(z_1, \dots, z_8) = \mathcal{S}[\prod_{i,j=1}^4 (z_i - z_{4+j})]$ , which is exactly the symmetric factor in (2). This is the unique lowest flux zero energy state of  $H_8$ . This argument for eight electrons is also true for arbitrary  $4n$  electrons because one can always think each term in the Hamiltonian (4) contains only one inter group electron pair. Therefore, the QGPf wave function is the ground state of the special Hamiltonian (4). The MR Pfaffian is also the zero energy state but has a higher flux.

*Quasiholes* — Since the wave function must be totally antisymmetric, similar to MR quasiholes in pairs[3, 18], the quasiholes create in quaternions, e.g., the 4-quasihole wave function is given by

$$\text{QGPf} \left( \frac{f(z_i, z_j, z_k, z_l; w_1, w_2, w_3, w_4)}{J(z_i, z_j, z_k, z_l)} \right), \quad (6)$$

where  $f(z_i, z_j, z_k, z_l; w_1, w_2, w_3, w_4) = (z_i - w_1)(z_j - w_2)(z_k - w_3)(z_l - w_4) + (ijkl)$  cycle. This is a zero energy state of the Hamiltonian (4) with the flux increasing to  $N_\phi = 2N - 2$  [19]. Note that if  $w_1 = w_2 = w_3 = w_4$ , it gives a Laughlin quasihole with charge  $1/2$ . Therefore, the quasihole charge is  $1/8$ . The  $4m$ -quasihole wave function can be defined in a similar way with

$$\begin{aligned} & f(z_i, z_j, z_k, z_l; w_1, \dots, w_{4m}) \\ &= (z_i - w_1) \cdots (z_i - w_m)(z_j - w_{m+1}) \cdots (z_j - w_{2m}) \\ & \times (z_k - w_{2m+1}) \cdots (z_k - w_{3m}) \\ & \times (z_l - w_{3m+1}) \cdots (z_l - w_{4m}) + (ijkl) \text{ cycle}. \end{aligned} \quad (7)$$

By exchanging the coordinates of quasiholes among four different sets  $(w_{am+1}, \dots, w_{(a+1)m})$  ( $a = 0, 1, 2, 3$ ), we can have  $C_{m-1}^{4m-1} C_{m-1}^{3m-1} C_{m-1}^{2m-1}$  states. However, these quasihole states are not all independent. For example, for  $m = 2$ , we have eight quasiholes and 35 different quasihole wave functions. A key relation to pick out the independent states reads [18]

$$[12]_i [34]_j - [14]_i [23]_j = x([12]_i [34]_j - [13]_i [24]_j), \quad (8)$$

where  $[12]_i = (z_i - w_1)(z_i - w_2)$ ,  $x = \frac{w_{13}w_{24}}{w_{14}w_{23}}$  and  $w_{12} = w_1 - w_2$ , etc. If we fix 4 in 8  $w_i$ , one can follow Ref. [18] step by step to check this relation is also correct when it is put into the QGPf. This means we have 3 different wave functions and only 2 of them are independent. If we fix two  $w_i$ , there are 15 different wave functions and 5-independent ones. For 35 different wave functions of total 8 quasiholes, there are 13 linearly independent. In general, according to exclusion statistics point of view [20], this degeneracy is equal to a generalized Fibonacci number  $F_{4m-3}^g$  which is defined by  $F_n^g = F_{n-1}^g + F_{n-2}^g + F_{n-3}^g$  with  $F_0^g = F_1^g = 1$  and  $F_2^g = F_1^g + F_0^g$ .  $F_1^g = 1$  and  $F_5^g = 13$  is consistent with  $m = 1$  and  $m = 2$  calculations.

There are three kinds of twisted states: (1) Take  $w_1 = 0$  and  $w_2 = w_3 = w_4 = \infty$  in the three quasihole wave

function; (2) Take  $w_1 = w_2 = 0$  and  $w_3 = w_4 = \infty$ ; (3) Take  $w_1 = w_2 = w_3 = 0$  and  $w_4 = \infty$ .

*Conformal Field Theory* — A unitary CFT may related to the QGPf state is a  $Z_2 \times Z_2$  parafermion theory with coset space  $SU(3)_2/U(1)^2$  and  $c = \frac{kD}{k+g} - 2 = 6/5$  ( $D = 8, k = 2, g = 3$ ) [13]. The  $Z_3$  parafermion theory with coset space  $SU(2)_3/U(1)$  supports the universal quantum computation [6]. Because of the level-rank duality between  $SU(3)_2$  and  $SU(2)_3$ , we expect the parafermion theory in coset space  $SU(3)_2/U(1)^2$  also supports the universal quantum computation.

There are three Majorana fermions  $\psi^\alpha$ ,  $\alpha = 1, 2, 3$ , which are parafermions graded by  $Z_2 \times Z_2$  (with the identity). The OPEs are given by

$$\begin{aligned} \psi^\alpha(z)\psi^\alpha(w) &= \frac{1}{z-w} + O((z-w)^0) \\ \psi^\alpha(z)\psi^\beta(w) &= \frac{c_{\alpha\beta}\psi^\gamma(w)}{(z-w)^{1/2}} + O((z-w)^{1/2}), \end{aligned} \quad (9)$$

where  $\alpha \neq \beta \neq \gamma$  in the second equation and  $c_{12} = c_{23} = c_{31} = e^{-i\pi/4}/\sqrt{2}$  and  $c_{\beta\alpha} = c_{\alpha\beta}^*$ . Three point parafermion correlation function for  $\alpha \neq \beta \neq \gamma$  is give by [13]

$$\langle \psi^\alpha(z_1)\psi^\beta(z_2)\psi^\gamma(z_3) \rangle = \frac{e^{-i\epsilon^{\alpha\beta\gamma}\pi/4}}{z_{12}^{1/2} z_{13}^{1/2} z_{23}^{1/2}} \quad (10)$$

Three twisted primary fields  $\sigma^{12}, \sigma^{23}, \sigma^{13}$  have conformal dimension  $1/10$ . When acting  $\psi^\alpha$  to  $\sigma^{\alpha\beta}$ , it behaves like the Ising spin field  $\sigma^\alpha$ . Using the OPEs, one has

$$\mathcal{A}\{\langle \mathcal{N}[\prod_{\alpha=1}^3 (\prod_{i=1}^4 \psi^\alpha(z_i))] \rangle\} = \frac{1}{J(z_1, \dots, z_4)}, \quad (11)$$

where  $\mathcal{N}$  is defined by subtracting the singularity from, e.g.,  $\psi^1(z_1)\psi^2(z_1)$ , etc. That is, let all  $\psi^2 = \psi^2(z+\epsilon)$  and  $\psi^3 = \psi^3(z+2\epsilon)$ . Then subtract the divergence with a term  $O(1/\epsilon^{1/2})$  and take  $\epsilon \rightarrow 0$  at the end of calculations. The normal ordering is :  $\psi^\alpha(z)\psi^\alpha(w) := \psi^\alpha(z)\psi^\alpha(w) - \frac{1}{z-w}$ , and so on. That is, we forbid the direct contraction between the same type Majorana fermions. Notice that this result is a branch cut-free generalization to (10). To get the QGPf, we calculate

$$\begin{aligned} \mathcal{A}\{\langle \mathcal{N}[\prod_{\alpha=1}^3 (\prod_{i=1}^N \psi^\alpha(z_i))] \rangle\} &= \text{QGPf}(G_4) \\ &+ \mathcal{A}[G_4^{N-8}G_3G_5 + G_4^{N-12}(G_5G_7 + G_3G_9) + \dots], \end{aligned} \quad (12)$$

where  $G_3 = 1/J(z_i, z_j, z_k)$ ;  $G_4^s = 1/[J(z_{i_1}, z_{i_2}, z_{i_3}, z_{i_4}) \cdots J(z_{i_{s-3}}, z_{i_{s-2}}, z_{i_{s-1}}, z_{i_s})]$ ;  $G_{2a+1}(z_1, \dots, z_{2a+1}) = 1/[z_{12}z_{23} \cdots z_{2a+1,1}z_{13}^{1/2} \cdots z_{2a,1}^{1/2}z_{2a+1,2}^{1/2}]$  for  $2a+1 \geq 5$ . All terms in square brackets on the right side include branch cut factors which can not be cancelled by multiplying a Jastrow factor  $[J(z_1, \dots, z_N)]^{p/q}$ . Projecting them away implies projecting to the LLL.

Therefore, the lowest Landau level projection leaves the QGPf only.

We notice that not all quasihole wave functions (7) can be fallen under a correlations function of this CFT. Using the twisted primary field what we can get is the following correlation function:

$$\mathcal{A}\{\langle \sigma^{12}(w_1)\sigma^{12}(w_2)\sigma^{12}(w_3)\sigma^{12}(w_4)\mathcal{N}[\prod_{\alpha=1}^3 : \prod_{i=1}^N \psi^\alpha(z_i) :] \rangle\}_{LL} \sim \text{QGPf}\left(\frac{f_4(z_i, z_j, z_k, z_l; w_1, w_2, w_3, w_4)}{J(z_i, z_j, z_k, z_l)}\right),$$

where  $f_4 = (z_i - w_1)^4(z_j - w_2)^4(z_k - w_3)^4(z_l - w_4)^4 + (ijkl)$  cycle. This is a 16 quasihole wave function with four at the same position.

*Edge Excitations* – The edge excitations can also be discussed by a parallel way to those in the Pfaffian state [21]. The Laughlin-type charge edge excitations are exactly the same as those in the Pfaffian state, which can be obtained by timing symmetric polynomials to the QGPf state. However, the neutral edge excitations are very different from those of the Pfaffian state, which are given by replacing the QGPf state by

$$\mathcal{A}(z_1^{p_1} \dots z_{4m'}^{p_{4m'}} \frac{1}{J(z_{4m'+1}, z_{4m'+2}, z_{4m'+3}, z_{4m'+4})} \dots) \quad (13)$$

These edge states gain the momentum  $\Delta M = \sum_{i=1}^{4m'} (p_i + 3/2)$ , instead of  $\sum_i (p_i + 1/2)$  for Majorana fermions.

For twisted states, the edge excitations are given by  $\mathcal{A}(z_1^{p_1} \dots z_{4m'}^{p_{4m'}} \frac{z_{4m'+1} z_{4m'+2} z_{4m'+3} z_{4m'+4}}{J(z_{4m'+1}, z_{4m'+2}, z_{4m'+3}, z_{4m'+4})} \dots)$  with  $\Delta M = \sum_{i=1}^{4m'} (p_i + 13/8)$  and other two raise momentum  $\Delta M = \sum_{i=1}^{4m'} (p_i + 7/4)$  and  $\Delta M = \sum_{i=1}^{4m'} (p_i + 15/8)$ .

*Experimental Implication* – Experimentally, the charge of the quasiparticle may be measured by the shot noise in a point contact tunnelling experiment, as measuring the fractional charge of the Laughlin quasiparticle [22]. For the MR Pfaffian state, the quasihole charge is  $\frac{e}{4}$  while it is  $\frac{e}{8}$  for the QGPf state. Recent proposed quasiparticle interferometry may measure the non-abelian statistics of the quasiparticles [23]. The different non-abelian statistical property will be reflected in this kind of experiments.

*Conclusions* — We have constructed a competing wave function of four-electron cluster in  $\nu = 5/2$ , the quaternary generalization of the pairing of composite fermions. This incompressible liquid state may challenge the MR Pfaffian state. The corresponding special Hamiltonian and the CFT were studied. The conformal field related to this QGPf state is dual to that of the Read-Rezayi quasihole at  $\nu = 13/5$ . Therefore, we expect a universal quantum computation in  $\nu = 5/2$ . The finite electron calculations with powerful computational methods are definitely required to compare with the MR Pfaffian. Because the system is particle-hole symmetric for

the Landau level mixing is neglected, an anti-QGPf state is expected like the anti-Pfaffian state competing to the Pfaffian state [24].

The author thanks Boris Noyvert, Zhenghan Wang, Xiao-Gang Wen and Zhongyuan Zhu for useful discussions. This work was supported in part by Chinese NNSF, the national program for basic research of MOST of China and a fund from CAS.

*Note added* The wave function (3) may be a member of a class of possible FQH wave functions recently proposed by Wen and Wang [25].

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